

## Index Laws

### Part 1

Simplifying Expressions

a  $x^8 \times x^{-6}$

b  $y^{-2} \times y^{-4}$

c  $6p^3 \div 2p^7$

d  $(2x^{-4})^3$

e  $y^3 \times y^{-\frac{1}{2}}$

f  $2b^{\frac{2}{3}} \times 4b^{\frac{1}{3}}$

g  $x^{\frac{3}{5}} \div x^{\frac{1}{5}}$

h  $a^{\frac{1}{2}} \div a^{\frac{4}{3}}$

i  $p^{\frac{1}{4}} \div p^{-\frac{1}{5}}$

j  $(3x^{\frac{2}{3}})^2$

k  $y \times y^{\frac{5}{6}} \times y^{-\frac{3}{2}}$

l  $4t^{\frac{3}{2}} \div 12t^{\frac{1}{2}}$

m  $\frac{b^2 \times b^{\frac{1}{4}}}{b^{\frac{1}{2}}}$

n  $\frac{y^{\frac{1}{2}} \times y^{\frac{1}{3}}}{y}$

o  $\frac{4x^{\frac{2}{3}} \times 3x^{-\frac{1}{6}}}{6x^{\frac{3}{4}}}$

p  $\frac{2a \times a^{\frac{3}{4}}}{8a^{-\frac{1}{2}}}$

### Part 2

Find the value of  $x$  such that

a  $2^x = 64$

b  $5^{x-1} = 125$

c  $3^{x+4} - 27 = 0$

d  $8^x - 2 = 0$

e  $3^{2x-1} = 9$

f  $16 - 4^{3x-2} = 0$

g  $9^{x-2} = 27$

h  $8^{2x+1} = 16$

i  $49^{x+1} = \sqrt{7}$

j  $3^{3x-2} = \sqrt[3]{9}$

k  $(\frac{1}{6})^{x+3} = 36$

l  $(\frac{1}{2})^{3x-1} = 8$

### Part 3

Solving Equations

a  $2^{x+3} = 4^x$

b  $5^{3x} = 25^{x+1}$

c  $9^{2x} = 3^{x-3}$

d  $16^x = 4^{1-x}$

e  $4^{x+2} = 8^x$

f  $27^{2x} = 9^{3-x}$

g  $6^{3x-1} = 36^{x+2}$

h  $8^x = 16^{2x-1}$

i  $125^x = 5^{x-3}$

j  $(\frac{1}{3})^x = 3^{x-4}$

k  $(\frac{1}{2})^{1-x} = (\frac{1}{8})^{2x}$

l  $(\frac{1}{4})^{x+1} = 8^x$

#### Part 4

##### Problem Solving

Q1

- a** The area of a rectangle is  $8y^5z^7$  and its length is  $4y^2z^3$ . Work out its width.
- b** Explain why the area of a rectangle of sides  $\sqrt[3]{8m^{-3}n^6}$  and  $\sqrt{16m^2n^{-4}}$  is independent of  $m$  and  $n$ . What is the area?

Q2

A cyclist travels  $4b^2c^{\frac{1}{2}}$  miles in  $3b^2c$  hours.  
What is her average speed?

Q3

A disc of radius  $3v^2z^{-2}$  cm is removed from a disc of radius  $4v^2z^{-2}$  cm. What is the remaining area?

Q4

The kinetic energy of a body is given by

$$\frac{1}{2} \times \text{mass} \times \text{velocity}^2$$

Work out the kinetic energy when mass =  $m$   
and velocity =  $9x^{\frac{3}{4}}c^{\frac{3}{4}}$

Q5

In an electrical circuit, the total resistance of two resistors,  $t_1$  and  $t_2$ , connected in parallel, is found by dividing the product of their resistances by the sum of their resistances.

Work out the total resistance when

$$t_1 = 3rs^2 \text{ ohms and } t_2 = 5rs^2 \text{ ohms.}$$

## Index Laws Solutions

### Part 1

Simplifying Expressions

$$a = x^2$$

$$b = y^{-6}$$

$$c = 3p^{-4}$$

$$d = 8x^{-12}$$

$$e = y^{\frac{5}{2}}$$

$$f = 8b^{\frac{2}{3} + \frac{1}{4}} = 8b^{\frac{11}{12}}$$

$$g = x^{\frac{2}{5} - \frac{1}{3}} = x^{\frac{1}{15}}$$

$$h = a^{\frac{1}{2} - \frac{4}{3}} = a^{-\frac{5}{6}}$$

$$i = p^{\frac{1}{4} - (-\frac{1}{5})} = p^{\frac{9}{20}}$$

$$j = 9x^{\frac{4}{5}}$$

$$k = y^{1 + \frac{5}{6} - \frac{2}{3}} = y^{\frac{1}{2}}$$

$$l = \frac{1}{3}t$$

$$m = b^{2 + \frac{1}{4} - \frac{1}{2}} = b^{\frac{7}{4}}$$

$$n = y^{\frac{1}{2} + \frac{1}{3} - 1} = y^{-\frac{1}{6}}$$

$$o = 2x^{\frac{2}{3} + (-\frac{1}{8}) - \frac{1}{4}} = 2x^{-\frac{1}{4}}$$

$$p = \frac{1}{4}a^{1 + \frac{2}{3} - (-\frac{1}{2})} = \frac{1}{4}a^{\frac{9}{2}}$$

### Part 2

$$a \quad 2^x = 2^6 \\ x = 6$$

$$b \quad 5^{x-1} = 5^3 \\ x - 1 = 3 \\ x = 4$$

$$c \quad 3^{x+4} = 27 = 3^3 \\ x + 4 = 3 \\ x = -1$$

$$d \quad (2^3)^x = 2^{3x} = 2 \\ 3x = 1 \\ x = \frac{1}{3}$$

$$e \quad 3^{2x-1} = 3^2 \\ 2x - 1 = 2 \\ x = \frac{3}{2}$$

$$f \quad 16 = 4^2 = 4^{3x-2} \\ 2 = 3x - 2 \\ x = \frac{4}{3}$$

$$g \quad (3^2)^{x-2} = 3^{2x-4} = 3^3 \\ 2x - 4 = 3 \\ x = \frac{7}{2}$$

$$h \quad (2^3)^{2x+1} = 2^{6x+3} = 2^4 \\ 6x + 3 = 4 \\ x = \frac{1}{6}$$

$$i \quad (7^2)^{x+1} = 7^{2x+2} = 7^{\frac{1}{2}} \\ 2x + 2 = \frac{1}{2} \\ x = -\frac{3}{4}$$

$$j \quad 3^{3x-2} = (3^2)^{\frac{1}{3}} = 3^{\frac{2}{3}} \\ 3x - 2 = \frac{2}{3} \\ x = \frac{8}{9}$$

$$k \quad (6^{-1})^{x+3} = 6^{-x-3} = 6^2 \\ -x - 3 = 2 \\ x = -5$$

$$l \quad (2^{-1})^{3x-1} = 2^{1-3x} = 2^3 \\ 1 - 3x = 3 \\ x = -\frac{2}{3}$$

### Part 3

Solving Equations

$$a \quad 2^{x+3} = (2^2)^x = 2^{2x} \\ x + 3 = 2x \\ x = 3$$

$$b \quad 5^{3x} = (5^2)^{x+1} = 5^{2x+2} \\ 3x = 2x + 2 \\ x = 2$$

$$c \quad (3^2)^{2x} = 3^{4x} = 3^{x-3} \\ 4x = x - 3 \\ x = -1$$

$$d \quad (4^2)^x = 4^{2x} = 4^{1-x} \\ 2x = 1 - x \\ x = \frac{1}{3}$$

$$e \quad (2^2)^{x+2} = (2^3)^x \\ 2^{2x+4} = 2^{3x} \\ 2x + 4 = 3x \\ x = 4$$

$$f \quad (3^3)^{2x} = (3^2)^{3-x} \\ 3^{6x} = 3^{6-2x} \\ 6x = 6 - 2x \\ x = \frac{3}{4}$$

$$g \quad 6^{3x-1} = (6^2)^{x+2} \\ 6^{3x-1} = 6^{2x+4} \\ 3x - 1 = 2x + 4 \\ x = 5$$

$$h \quad (2^3)^x = (2^4)^{2x-1} \\ 2^{3x} = 2^{8x-4} \\ 3x = 8x - 4 \\ x = \frac{4}{5}$$

$$i \quad (5^3)^x = 5^{x-3} \\ 5^{3x} = 5^{x-3} \\ 3x = x - 3 \\ x = -\frac{3}{2}$$

$$j \quad (3^{-1})^x = 3^{x-4} \\ 3^{-x} = 3^{x-4} \\ -x = x - 4 \\ x = 2$$

$$k \quad (2^{-1})^{1-x} = (2^{-3})^{2x} \\ 2^{x-1} = 2^{-6x} \\ x - 1 = -6x \\ x = \frac{1}{7}$$

$$l \quad (2^{-2})^{x+1} = (2^3)^x \\ 2^{-2x-2} = 2^{3x} \\ -2x - 2 = 3x \\ x = -\frac{2}{5}$$

#### Part 4

##### Problem Solving

Q1

**a**  $2y^3z^4$

**b** Area = 8. Therefore the area is independent of  $m$  and  $n$

Q2

$$\frac{4}{3c^{\frac{1}{2}}} \text{ mph}$$

Q3

$$7\pi v^4 z^{-4} \text{ cm}^2$$

Q4

$$\frac{81}{2} m x^{\frac{3}{2}} c^{\frac{3}{2}}$$

Q5

$$\frac{15}{8} r s^2 \text{ ohms}$$

(Full worked solutions on Kerboodle Ex1.2B Q 5,6,8,11,13)